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Total No. of Questions : 09

B.Tech.(2008-2010 Batches) (Sem.–1)

ENGINEERING MATHEMATICS–I

Subject Code : AM-101

Paper ID : [A0111]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION - A

l. Solve the following :

(a) State De-Moivre's theorem.

(b) Prove that $i^i = e^{-(4n+1)\frac{\pi}{2}}$.

(c) Define Cauchy's Integral Test.

(d) Show that $\int_0^\infty \frac{1}{x^{7/4}} e^{-\sqrt{x}}\, dx = \frac{8}{3}\sqrt{\pi}$.

(e) Write an equation for ellipsoid.

(f) If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$. Find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$

(g) Find the equation of the tangent plane and the normal to $xyz = a^2$ at (x_1, y_1, z_1).

(h) Find the area of the circle of radius r.

(i) Define Uniform Convergence.

(j) Expand $x^2y + 3y - 2$ in powers of $x - 1$ and $y + 2$ using Taylor's theorem.

SECTION - B

2. Trace the curve, $y^2 = (x + 1)^3$.

3. Find the whole length of the curve, $x^{2/3} + y^{2/3} = a^{2/3}$.

4. If $u = \operatorname{cosec}^{-1}\left(\dfrac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}\right)^{1/2}$, then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right).$$

5. Find the point on the surface $z^2 = xy + 1$ nearest to the origin.

SECTION - C

6. Find the equation of the cone whose vertex is at origin and guiding curve is

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{1} = 1, x + y + z = 1.$$

7. Find the volume of the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

8. Test the convergence of the series $\dfrac{x}{1} + \dfrac{1}{2} \cdot \dfrac{x^3}{3} + \dfrac{1}{2} \cdot \dfrac{3}{4} \cdot \dfrac{x^5}{5} + \ldots\ldots\ldots\ldots\infty$.

9. Prove that the *nth* roots of unity form the geometric progression. Also show that the sum of these n roots is zero and their product is $(-1)^{n-1}$.